The Laws of Logarithms

These notes are intended as a summary of section 5.5 (p. 388 – 392) in your workbook. You should also read the section for more complete explanations and additional examples.

Using your calculator, verify that each of the following equalities is true:

$$\log 2 + \log 3 = \log 6$$

$$\log 8 - \log 2 = \log 4$$

$$3\log 2 = \log 8$$

These three equalities can be generalized to form the three **laws of logarithms**.

The Product Law

$$\log_a (MN) = \log_a M + \log_a N \qquad \begin{cases} M > 0 \\ N > 0 \end{cases}$$

Proof

The Quotient Law

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \qquad \begin{cases} M > 0 \\ N > 0 \end{cases}$$

Proof

The Power Law

$$\log_a\left(M^n\right) = n\log_a M \qquad \begin{cases} M > 0 \\ n > 0 \end{cases}$$

Proof

Example 1 (sidebar p. 390)
Simplify each expression. Use a calculator to verify the answer.

a) $\log 7 + \log 8$

b) $5\log 2$

c) $\log 80 - \log 16$

Example 2 (sidebar p. 390)
Write each expression as a single logarithm.

a) $\log x + 3\log y$

b) $\log x + 2\log y - 4\log z$

c) $\log_2 6 - 3$

Example 3 (sidebar p. 391)
Write each expression in terms of $\log a$, $\log b$, and/or $\log c$.

a) $\log\left(\frac{a}{b^2}\right)$

b) $\log\left(\frac{a^2 b^{\frac{1}{3}}}{c}\right)$

Example 4 (sidebar p. 392)
Evaluate each expression.

a) $3\log_9 6 - \log_9 72$

b) $2\log_4 6 - 3\log_4 3 + \log_4 12$

Change of Base Formula

To use the LOG key on your calculator to evaluate a logarithm with base other than 10, the base of the logarithm must be changed to 10. This is accomplished using the change of base formula:

$$\log_b x = \frac{\log_a x}{\log_a b} \qquad \begin{cases} a,b,x > 0 \\ a,b \neq 1 \end{cases}$$

Proof

Example (not in workbook)
Evaluate each logarithm.

a) $\log_2 3$

b) $\log_7 3614$

c) $\log_6 423$

Homework: #4, 5, 8, 11 – 16, 18 in the section 5.5 exercises (p. 393 – 398). Answers on p. 399.
#3 – 5 in the section 5.6 exercises (p. 405 – 410). Answers on p. 411.